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APPROXIMATION CALCULATION OF RADIANT HEAT TRANSFER IN A DUCT OF RECTANGULAR CROSS SECTION

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Аннотация—Приводятся некоторые указания к приближенному расчету лучистого теплообмена в цилиндрической системе с прямоугольным поперечным сечением. В условиях, предусмотренных в работе, применима двухмерная теория. Рассматривается система из серых поверхностей, заполненная поглощающей средой, излучение в которой не отражается и не преломляется. Метод решения проблемы основывается на интегральных уравнениях. Расчет производится в следующем порядке:

1. Система распределеняется на зоны;
2. В случае канала с прямоугольным сечением при 4-х поверхностных и 2-х объемных зонах значения взаимных поверхностей и угловых коэффициентов прямого лучистого обмена вычисляются по формулам (21)–(29);
3. Значения разрешающих взаимных поверхностей и угловых коэффициентов вычисляются по формулам (30)–(33);
4. Результативные лучистые потоки (или температуры зон) определяются применением формул (17) и (18).

Метод иллюстрирован примером. Ход вычисления примера дан в табл. 3, а в табл. 4 сравниваются некоторые экспериментальные данные с исходными условиями и результатами численного примера. Метод может быть применен при решении проблем лучистого теплообмена с учетом расположения факела в топке. Метод является также полезным, если рассматривается распределение результативного лучистого потока в поперечном сечении топки.

NOMENCLATURE

A, B, C,	local points in radiating system, see Fig. 1;
a,	absorptivity of walls;
D,	determinant used in the example;
$E = 5{\cdot}77 \times 10^{-8}\ T^4$,	black-body radiation density, W/m^2;
F,	area of surface zone, m^2;
G,	volume–wall interchange area, defined by (15), m^2;
H,	wall–wall interchange area, defined by (12) and (13), m^2;
h,	height of the duct, see Fig. 3, m;
K, L,	expressions in equation (1), defined by (2), m^{-2};
k,	absorption coefficient of the medium, m^{-1};
l,	beam length, see Fig. 1 and equation (20), m;
M, N, S,	special functions in equations (21)–(24), values are given in Table 1;
P,	pressure of gas medium in (19), kg/cm^2;
Q,	heat flux, W;
q,	heat flux density, W/m^2;
r,	reflectivity of the walls;
S,	length of the duct, see (23) and (24), m;
T,	absolute temperature, °K;
t,	temperature, °C;
V,	volume, m^3;
x,	co-ordinate, Fig. 3, m;

y,	altitude of flame, Fig. 4, m;

Greek symbols

Γ,	functions of equations (4)–(10) defined by equations (5) and (9), m^{-2};
γ,	volume–wall interchange factor defined by (15);
η,	volume radiation density, defined by (3), W/m^3;
ϑ,	angle, see Fig. 1;
ρ,	geometrical length, defined by Fig. 3, m;
ψ,	wall–wall interchange factor, defined by (11) and (13).

Subscripts

i, k,	refers to surface zone;
j, v, w,	refers to volume zone;
m,	total number of surface zones;
n,	total number of volume zones;
u,	refers to surface or volume zone;
eff,	effective flux from the surface;
inc,	radiation flux incident on the surface;
res,	resulting flux through the surface;
1, 2, 3, 4	refers to surface zone;
1, 2,	subscripts at N used for separate kinds of functions of N.

INTRODUCTION

THE theory of radiant heat transfer in a system of surfaces with an absorbing medium has been well established theoretically [1–5], but for practical calculations of such systems the over-simplified method based on the application of the effective thickness of the radiating layer is still used in most cases. This situation is brought about by the complexity of calculation when the theory is applied. The method proposed by Hottel and Cohen [6] also demands much computing work. However in a number of furnaces and duct-like combustion chambers heat radiation and absorption of bottoms may be practically neglected. Conditions approximately correspond to the case when a radiating system is infinitely long in one dimension and the problem is therefore two-dimensional. Plainly considerable saving in computation can be achieved if a three-dimensional problem is reduced to a two-dimensional one.

The purpose of the paper is to suggest some methods involving the application of the two-dimensional theory to calculating radiant heat transfer in cylindrical systems of rectangular cross section. The following assumptions are made:

1. The process of radiant heat transfer takes place at conditions of local equilibrium.
2. The flow and heat conduction of gray gas medium are neglected.
3. The absorption coefficient of the medium is taken to be constant throughout the radiating system, and the reflectivity and refractivity of the medium are assumed to be zero.
4. The Lambert law is assumed applicable and all the surfaces of the radiating system are considered gray.

Since the present solution of the problem is reduced to the division of the radiating system into separate zones, more accurate results can be obtained, the less the optical density of the medium.

SOME THEORETICAL ASPECTS

The density of effective radiant flux leaving the point A (Fig. 1) can be expressed as the sum of the emitted flux density $a(A)\,E(A)$ and the reflected flux density $r(A)\,q_{inc}(A)$, where $a(A)$ and $r(A)$ are absorptivity and reflectivity of the walls at the point A, and $E(A)$ is the black-body radiation density of 1 m^2 of the surface at A. The flux density of incident radiation $q_{inc}(A)$ is the sum of the effective flux density, reaching A from the other points B of the walls, and of the flux density emitted by the radiating medium from points C. The equation of the effective flux density [1–5] with the present assumptions is as follows:

$$q_{inc}(A) = a(A)\,E(A) + r(A)\left[\int_{F_B} q_{eff}(B)\,K(AB)\,\mathrm{d}F_B + \int_V \frac{l}{4}\,\eta(C)\,L(AC)\,\mathrm{d}V\right] \tag{1}$$

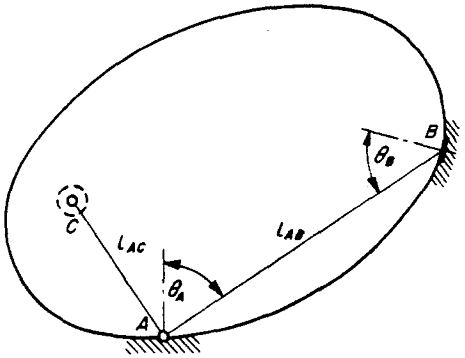

FIG. 1. Schematic representation of radiating system [equation (1)].

where

$$\left.\begin{aligned} K(AB) &= \frac{\cos\vartheta_A \cos\vartheta_B}{\pi l_{AB}^2} \exp(-kl_{AB}), \\ L(AB) &= \frac{\cos\vartheta_A}{\pi l_{AC}^2} \exp(-kl_{AC}) \end{aligned}\right\} \quad (2)$$

and k is the absorption coefficient of the medium, l is the length of the beam, ϑ is the angle between the beam and the normal to the wall.

For the volume radiation density $\eta(C)$ the well-known formula can be written:

$$\eta(C) = 4kE(C). \quad (3)$$

In the theory of integral equations the solution of equation (1) is given as follows:

$$q_{eff}(A) = a(A)\,E(A) + r(A)\,[\textstyle\int_V kE(C)\, L(AC)\,dV + \int_{F_B} a(B)\,E(B)\,\Gamma(AB)\,dF_B + \int_{F_B} r(B) \int_V kE(C)\,L(BC)\,\Gamma(AB)\,dV\,dF_B] \quad (4)$$

where the function $\Gamma(AB)$ of equation (1) is a solution of integral equations:

$$\left.\begin{aligned} \Gamma(AB) &= K(AB) + \textstyle\int_{F_B} r(B)\,K(PB)\,\Gamma(AP)\,dF_p \\ \Gamma(AB) &= K(AB) + \textstyle\int_{F_B} (B)\,K(AP)\,\Gamma(PB)\,dF_p. \end{aligned}\right\} \quad (5)$$

According to the second law of thermodynamics it follows from equation (4) that

$$\textstyle\int_V kL(AC)\,dV + \int_{F_B} a(B)\,\Gamma(AB)\,dF_B + \int_{F_B} r(B) \int_V kL(BC)\,\Gamma(AB)\,dV\,dF_B = 1. \quad (6)$$

Hence, according to equations (4) and (6) one obtains the resulting heat flux density at A:

$$q_{res}(A) = a(A)\,\{\textstyle\int_{F_B} [E(B) - E(A)]\, a(B)\,\Gamma(AB)\,dF_B + \int_V [E(C) - E(A)]\,[L(AC) + \int_{F_B} r(B)\,L(BC)\, \Gamma(AB)\,dF_B]\,k\,dV\}. \quad (7)$$

The solution of the problem is given in equations (5) and (7). It is interesting to obtain however an approximate solution which is more suitable for application to design and computation. It is usually realized by division of the system into zones with constant temperatures and physical properties. Thus if $E(C_j) = \text{const.}$ is assumed at each volume zone j, the volume integrals in equations (1)–(7) may be expressed by means of surface integrals taken over the whole surface F_j enclosing the whole zone volume V_j:

$$\textstyle\int_{V_j} kL(AC_j)\,dV_j = \oint_{F_j} k(AC_j)\,dF_j. \quad (8)$$

It would also serve the purpose to define $\Gamma(AC)$ as follows:

$$\Gamma(AC) = K(AC) + \textstyle\int_{F_B} r(B)\,K(BC)\,\Gamma(AB)\,dF_B \quad (9)$$

If the system consists of m surface zones F_i and n volume zones V_j, then equation (7) may be written separately for each surface zone k by means of equations (8) and (9):

$$q_{res}(A_k) = a_k\,\{\sum_{i=1}^{m} a_i\,[E(B_i) - E(A_k)] \textstyle\int_{F_i} \Gamma(A_k B_i)\,dF_i + \sum_{j=1}^{n} [E(C_j) - E(A_k)] \oint_{F_j} \Gamma(A_k C_j)\,dF_j\},$$
$$k = 1, 2, 3, \ldots, m. \quad (10)$$

Computing the resulting heat flux density

$q_{res}(A_k)$, the direct angle factors are applied, defined as

$$\psi_{ki} = \int_{F_i} K(A_k B_i)\, dF_i, \tag{11}$$

and the direct interchange areas defined as

$$H_{ki} = \int_{F_k} \psi_{ki}\, dF_k. \tag{12}$$

If the part of heat quantity transferred by multiple reflections is taken into consideration, the overall interchange factors and areas are defined as follows:

$$\left.\begin{aligned} \bar{\psi}_{ki} &= \int_{F_i} \Gamma(A_k B_i)\, dF_i, \\ \bar{H}_{ki} &= \int_{F_k} \bar{\psi}_{ki}\, dF_k. \end{aligned}\right\} \tag{13}$$

The leading angle factors and interchange areas are calculated by integration of equations (5).

When the radiating system is divided into zones, it is usually assumed that the factors ψ_{ki} may be approximately replaced by their average values H_{ki}/F_k. With this assumption equations (5) are reduced to a system of linear algebraic equations:

$$\left.\begin{aligned} &\left(\frac{F_i}{r_i} - H_{ii}\right)\frac{r_i}{F_i}\bar{\psi}_{ki} - \sum_{\substack{u=1\\u\neq i}}^{m} \frac{r_u}{F_u} H_{ui}\,\bar{\psi}_{ku} = \psi_{ki}, \\ &\left(\frac{F_i}{r_i} - H_{ii}\right)\frac{r_i}{F_i}\bar{H}_{ki} - \sum_{\substack{u=1\\u\neq i}}^{m} \frac{r_u}{F_u} H_{ui}\,\bar{H}_{ku} = H_{ki},\ i, k = 1, 2, 3, \ldots, m. \end{aligned}\right\} \tag{14}$$

The volume–wall interchange factors and areas can be expressed as

$$\left.\begin{aligned} \gamma_{kj} &= \oint_{F_j} K(A_k C_j)\, dF_j, \\ \bar{\gamma}_{kj} &= \oint_{F_j} \Gamma(A_k C_j)\, dF_j, \\ G_{kj} &= \int_{F_k} \gamma_{kj}\, dF_k, \\ \bar{G}_{kj} &= \int_{F_k} \bar{\gamma}_{kj}\, dF_k. \end{aligned}\right\} \tag{15}$$

Assuming γ_{uj} may be approximately replaced by their average values G_{ui}/F_u we may calculate values of γ_{kj} and G_{kj} from algebraic equations. Then equations (9) can be replaced by the system of algebraic equations:

$$\left.\begin{aligned} \bar{\gamma}_{kj} &= \gamma_{kj} + \sum_{u=1}^{m} \frac{r_u}{F_u} G_{uj}\,\bar{\psi}_{ku}, \\ \bar{G}_{kj} &= G_{kj} + \sum_{u=1}^{m} \frac{r_u}{F_u} G_{uj}\,\bar{H}_{ku}, \\ k &= 1, 2, 3, \ldots, m; \quad j = 1, 2, 3 \ldots, n. \end{aligned}\right\} \tag{16}$$

Proceeding from this we may recommend the following procedure for radiative heat-transfer calculation:

(1). The system is divided into surface and volume zones. The division should be made in such a way that each zone is characterized by constant temperature and physical properties. In the general case the solutions of algebraic equations (14) and (16) are not exactly the solutions of integral equations (5) and (9), the accuracy of results increasing with increase in the number of zones. For approximate calculation the smallest possible number of zones is desirable since this considerably reduces computation.

(2). The direct interchange factors and areas $\bar{\psi}_{ki}$, H_{ki}, γ_{kj}, G_{ki} are calculated. Some of the values of ψ_{ki} and H_{ki} are determined directly while others are obtained from application of symmetry $H_{ki} = H_{ik}$ and the property of being able to add integrals $H_{ki} + H_{ku} = H_{k,\,i+u}$ and $\psi_{ki} + \psi_{ku} = \psi_{k,\,i+u}$. Calculation of γ_{kj} or G_{kj} is reduced to determination of the values of ψ_{ki} or H_{kj} and to setting up their algebraic sums according to equation (15). If the distance of the point A_k from the surface F_j becomes zero, $\psi_{kj} = 1$, and at zero distance between parallel surfaces F_k and F_j, $H_{kj} = F_k$ if $F_k > F_j$, and $H_{kj} = F_j$ if $F_k > F_j$.

(3). The values of overall interchange factors and areas $\bar{\psi}_{ki}$, $\bar{H}_{ki}$, $\bar{\gamma}_{kj}$, $\bar{G}_{ki}$ are obtained by solving equations (14) and (16).

(4). Equation (10) of the resulting heat flux density at A_k can be written in the form:

$$q_{res}(A_k) = a_k \left\{\sum_{i=1}^{m} a_i\,[E(B_i) - E(A_k)]\,\bar{\psi}_{ki} + \sum_{j=1}^{n} [E(C_j) - E(A_k)]\,\bar{\gamma}_{kj}\right\} \tag{17}$$

and the whole resulting heat quantity received or emitted by the surface F_k, is obtained by integrating function (17) over the surface F_k:

$$Q_{res,}\, k = a_k \left\{ \sum_{i=1}^{m} a_i \left[E(B_i) - E(A_k)\right] \bar{H}_{ki} + \sum_{j=1}^{n} \left[E(C_j) - E(A_k)\right] \bar{G}_{kj} \right\}. \qquad (18)$$

It should be noted that the majority of assumptions used in the present method are hardly relevant since most of the initial data are known only approximately and usually results of high accuracy are not demanded. Possibly an assumption of prime importance is one of gray radiation of gas medium. If selective absorbtivity and emissivity of gas medium are taken into account, then a correct solution of the problem in the case of gray walls becomes very complicated and some results are obtained only for a plane gas layer [7]. However it is shown [8] that radiation of gas medium may be approximately considered gray. If the combustion products of the fuel are taken as a "gray" gas medium, the value of the absorption coefficient can be calculated by the formula recommended in [8]:

$$k = (1 - 0{\cdot}38 \times 10^{-3}T)\left(0{\cdot}8 + 1{\cdot}6\frac{P_{H_2O}}{P}\right) \Big/ \sqrt{\left(\frac{P_{CO_2} + P_{H_2O}}{l}\right)} \qquad (19)$$

where T is the medium temperature. To determine the mean beam length l, the formula of the characteristic size may be recommended:

$$l = \frac{4V}{F}, \text{ m} \qquad (20)$$

where F is determined as the whole area of surfaces enclosing the volume V of the radiant system.

Calculation is more complicated but gives greater accuracy if the gas medium is considered gray in one part of the spectrum and transparent in the other part [6, 9]. Application of this rule becomes more complicated if radiation of solid dust particles in a gas medium is to be allowed for.

A DUCT OF RECTANGULAR CROSS SECTION

Radiation in a system having the form of a cylindrical duct of rectangular cross section will be considered next. The problem is reduced to the two-dimensional case, if the temperature field and the physical properties along the length of the duct are assumed to be constant and the radiation effects of the bottoms are not taken into account. The schedule of calculation of radiant heat transfer is given in the previous section. Additional recommendations are presented as follows:

(1). In principle there is no difficulty in taking any number of zones, but computation is rarely successful without experimental data on temperature distribution. It is therefore assumed that there are two volume zones only, as shown in Fig. 2. The volume can be divided into two

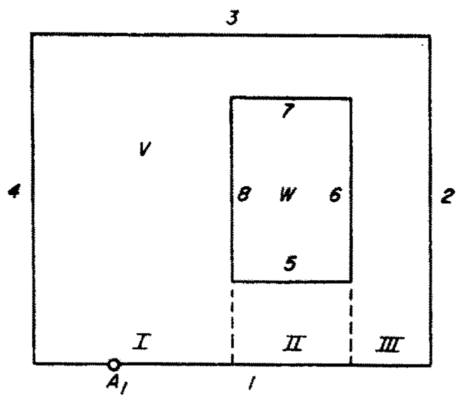

FIG. 2. Rectangular cross section of duct. 1–4—surface zones, v and w—volume zones.

zones by taking for the first zone the part of the volume filled with flame and for the second zone the part of the volume filled with non-combustible gases.

(2). Values of direct interchange factors and areas should be determined for the four geometrical cases shown in Fig. 3. The values can be taken from the tables in [10] (which present values with the maximum error of 0·5 per cent) or they may be calculated in an approximate analytical way [11]. In the later case it is possible that the maximum error may approach 2 per cent of the total sum of heat quantities, absorbed and passing through the medium. In that case however tables or diagrams of special functions only of one argument (optical density of the medium) are necessary. Direct interchange

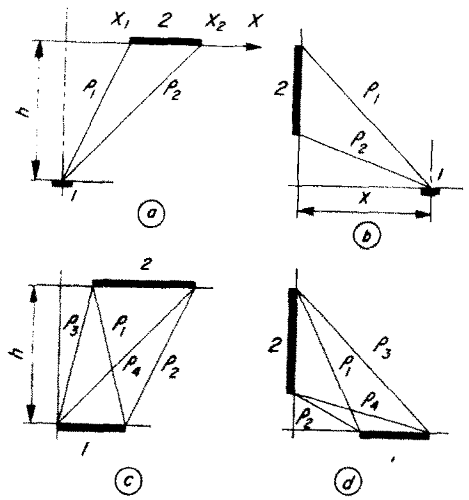

FIG. 3. Cross sections for calculation of direct interchange factors and areas: *a*—equation (21), *b*—equation (22), *c*—equation (23), *d*—equation (24).

factors and areas of cross sections given in Fig. 3 can be evaluated as follows:

$$\left.\begin{aligned}&\text{(a)}\quad \psi_{12} = \psi_{1x_2} - \psi_{1x_1},\\ &\psi_{1x} = \frac{x}{2\rho}\left\{M(kh) + [S(kh) - M(kh)]\left(\frac{x}{\rho}\right)^2\right\}\end{aligned}\right\} \quad (21)$$

$$\text{(b)}\quad \psi_{12} = \frac{x}{2}\left[\frac{N_1(k\rho_2)}{\rho_2} - \frac{N_1(k\rho_1)}{\rho_1}\right] \quad (22)$$

$$\text{(c)}\quad H_{12} = \frac{s}{2}\left\{[\rho_1 + \rho_4 - \rho_2 - \rho_3]\, S(kh)\left(\frac{1}{\rho_1} + \frac{1}{\rho_4} - \frac{1}{\rho_2} - \frac{1}{\rho_3}\right)[S(kh) - M(kh)]\, h^2\right\} \quad (23)$$

$$\text{(d)}\quad H_{12} = \frac{s}{2k}[N_2(k\rho_2) + N_2(k\rho_3) - N_2(k\rho_1) - N_2(k\rho_4)], \quad (24)$$

where s is the length of the duct.

Special functions of optical density of the medium $M(Z)$, $N_1(Z)$, $N_2(Z)$ and $S(Z)$ were defined in a previous paper [11] and values of these functions are compiled in Table 1. It should be noted however that a better accuracy of interpolation will be obtained when using logarithms of the functions also given in Table 1.

The volume–wall interchange factors γ_{kw} are evaluated as sums of ψ_{ki} according to equation (15):

$$\gamma_{kw} = \sum_{i=5}^{8} \pm \psi_{ki}, \quad k = 1, 2, 3, 4. \quad (25)$$

In determining the sign of ψ_{ki} it should be taken into consideration that every ray crosses the wall enclosing the volume w (Fig. 2) in two points. The interchange factor from A_k to the surface is taken positive for the nearest point of intersection and negative for the other point of intersection. So for the point A_1 in Fig. 2:

$$\gamma_{1w} = \psi_{15} + \psi_{18} - \psi_{16} - \psi_{17} \quad (25a)$$

If additivity of integrals is taken into account, γ_{kv} can be evaluated by the formula:

$$\gamma_{kv} = 1 - \gamma_{kw} - \sum_{\substack{i=1\\ i\neq k}}^{4} \psi_{ki}, \quad k = 1, 2, 3, 4. \quad (26)$$

In working our volume–wall interchange areas G_{kw}, the dependence of the sign upon the location of the point A_k must be again taken into account. For example dividing the surface 1 into three regions I, II and III as shown in Fig. 2, we obtain

$$\left.\begin{aligned} G_{1\mathrm{I}w} &= H_{1\mathrm{I}5} - H_{1\mathrm{I}6} - H_{1\mathrm{I}7} + H_{1\mathrm{I}8},\\ G_{1\mathrm{II}w} &= H_{1\mathrm{II}5} - H_{1\mathrm{II}6} - H_{1\mathrm{II}7} - H_{1\mathrm{II}8},\\ G_{1\mathrm{III}w} &= H_{1\mathrm{III}5} + H_{1\mathrm{III}6} - H_{1\mathrm{III}7} - H_{1\mathrm{III}8}\end{aligned}\right\} \quad (27)$$

and by adding equations (27):

$$G_{1w} = H_{15} - H_{17} + H_{1\mathrm{III}6} + H_{1\mathrm{I}8} - H_{(1\mathrm{I}+1\mathrm{II})6} - H_{(1\mathrm{II}+1\mathrm{III})8}. \quad (28)$$

The same principle is used for evaluating other values of G_{kw}.

The values of G_{kv} are calculated from the formula:

$$G_{kv} = F_k - G_{kw} - \sum_{\substack{i=1\\ i\neq k}}^{4} H_{ki}, \quad k = 1, 2, 3, 4. \quad (29)$$

(3). The overall interchange factors and areas are estimated according to (14) and (16). Under given conditions we have:

Table 1. Some values of the special functions $M(Z)$, $N_1(Z)$, $N_2(Z)$ and $S(Z)$

Z	$10^4\,M(Z)$	$10^4\,N_1(Z)$	$10^4\,N_2(Z)$	$10^4\,S(Z)$	$\log_{10}(10^4\,M)$	$\log_{10}(10^4\,N_1)$	$\log_{10}(10^4\,N_2)$	$\log_{10}(10^4\,S)$
0	10000	10000	4244	10000	4·0000	4·0000	3·6278	4·000
0·1	8818	6826	3433	8326	3·9454	3·8342	3·5357	3·9204
0·2	7793	5241	2836	7039	3·8917	3·7194	3·4524	3·8475
0·3	6899	4163	2368	6001	3·8388	3·6194	3·3744	3·7783
0·4	6116	3372	1994	5146	3·7865	3·5279	3·2996	3·7115
0·5	5429	2767	1688	4432	3·7347	3·4420	3·2274	3·6466
0·6	4823	2293	1436	3821	3·6832	3·3604	3·1571	3·5833
0·7	4289	1915	1226	3321	3·6323	3·2821	3·0885	3·5213
0·8	3816	1608	1050	2886	3·5816	3·2064	3·0214	3·4603
0·9	3399	1358	903	2514	3·5313	3·1329	2·9555	3·4004
1·0	3028	1152	777	2194	3·4812	3·0613	2·8906	3·3412
1·1	2700	980	671	1918	3·4314	2·9912	2·8268	3·2827
1·2	2408	837	580	1679	3·3818	2·9226	2·7638	3·2250
1·3	2150	717	503	1472	3·3323	2·8553	2·7016	3·1678
1·4	1919	615	436	1292	3·2831	2·7890	2·6400	3·1111
1·5	1714	529	379	1135	3·2341	2·7237	2·5791	3·0548
1·6	1532	456	330	998	3·1852	2·6594	2·4774	2·9992
1·7	1369	394	288	879	3·1365	2·5957	2·4591	2·9439
1·8	1224	341	251	774	3·0879	2·5330	2·3999	2·8889
1·9	1095	296	219	683	3·0395	2·4708	2·3410	2·8343
2·0	980	257	192	603	2·9912	2·4094	2·2826	2·7802
2·5	564	129	99	326	2·7514	2·1099	1·9965	2·5131
3·0	326	66	52	179	2·5138	1·8215	1·7185	2·2519
3·5	190	35	28	99	2·2783	1·5403	1·4456	1·9952
4·0	111	18	15	55	2·0445	1·2648	1·1790	1·7419
4·5	65	10	8	31	1·8116	1·0000		1·4914
5·0	38	5	4	18	1·5798			1·2455
5·5	22	3	2	10	1·3502			1·0000
6·0	13	2	1	6	1·1206			
6·5	8	1	1	3				
7·0	5			2				
7·5	3			1				
8·0	2			1				

$$\bar{\psi}_{ki} - \sum_{\substack{u=1\\u\neq i}}^{4} \frac{r_u}{F_u} H_{ui}\,\bar{\psi}_{ku} = \psi_{ki,\,i}, \quad k = 1, 2, 3, 4. \tag{30}$$

$$\bar{H}_{ki} - \sum_{\substack{u=1\\u\neq i}}^{4} \frac{r_u}{F_u} H_{ui}\,\bar{H}_{ku} = H_{ki,\,i}, \quad k = 1, 2, 3, 4. \tag{31}$$

$$\bar{\gamma}_{kj} = \gamma_{kj} + \sum_{i=1}^{4} \frac{r_i}{F_i} G_{ij}\,\bar{\psi}_{ki}, \quad k = 1, 2, 3, 4; \quad j = v, w. \tag{32}$$

$$\bar{G}_{kj} = G_{kj} + \sum_{i=1}^{4} \frac{r_i}{F_i} G_{ij}\,\bar{H}_{ki}, \quad k = 1, 2, 3, 4; \quad j = v, w. \tag{33}$$

Equations (30) and (32) are to be used if the distribution of local radiant flux density is wanted. They may be applied also if the temperature distribution of the walls is to be estimated. The number of calculation operations is dependent on the number of the points on the wall "k" chosen for calculation.

By using equations (31), calculation is reduced because of the condition $\bar{H}_{ki} = \bar{H}_{ik}$. If the volume zones are located symmetrically a further reduction of computation is possible as shown in the following numerical example.

(4) Final results are obtained by use of (17) and (18) where $i, k = 1, 2, 3, 4$ and $j = v, w$.

NUMERICAL EXAMPLE

A furnace chamber of rectangular cross section (shown in Fig. 4) is examined. The chamber is considered to be a long cylindrical duct, so the problem can be solved as a two-dimensional one. It is assumed that the radiant heat transfer proceeds from the gas medium to the bottom of the duct and the other walls are to be taken as adiabatic surfaces with zero resulting flux. The content of carbon dioxide in the combustion products of the gas fuel is 8·2 per cent and that of the vapor is 17·8 per cent. Using (19) we find that with the characteristic size of the duct of 0·57 m [according to (20)] and with the given partial pressures of carbon dioxide and water vapor, the value of the absorption coefficient of the gas medium is $k = 0{\cdot}364\ \mathrm{m}^{-1}$. Some further information about the physical properties and technological data are given in Table 2.

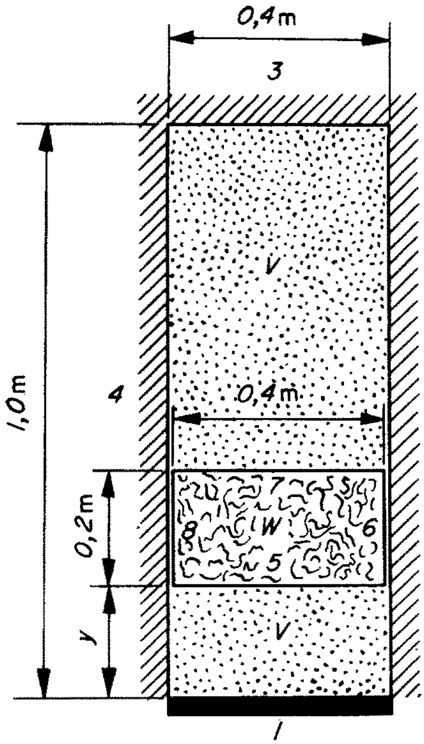

FIG. 4. Cross section of furnace chamber for numerical example: 1—bottom of furnace; 2, 3, 4—adiabatic walls of furnace; v—non-combustible gases; w—flame zone.

Our task is to determine the influence of various locations of burners on radiant heat transfer. Consider two cases:

1. Flame comes into contact with the bottom of the duct, $y = 0$.
2. Between the flame and the bottom there exists a gas layer of a thickness of $y = 0{\cdot}7$ m.

The cross section of the flame is taken to be rectangular with a thickness of 0·2 m and width equal to that of the chamber.

Before solving this example it must be noted that the existence of adiabatic surfaces in the system leads to some modifications of the problem. Since the system is symmetrical, it can be taken that $E_2 = E_4$, $\bar{H}_{12} = \bar{H}_{14}$ and $\bar{H}_{23} = \bar{H}_{21} = \bar{H}_{12}$. The resulting heat flux at the bottom is given as:

$$Q_{res,1} = a\,[(E_v - E_1)\,\bar{G}_{1v} + (E_w - E_1)\,\bar{G}_{1w} + 2\,(E_2 - E_1)\,a\bar{H}_{12} + (E_3 - E_1)\,\bar{H}_{13}a] \quad (34)$$

and the unknown values of $E_2 - E_1$ and $E_3 - E_1$ are determined from the condition that the resulting heat fluxes are zero at adiabatic walls of the furnace:

$$\left.\begin{aligned} &\bar{G}_{2v}(E_v - E_2) + \bar{G}_{2w}(E_w - E_2) + a\bar{H}_{12} (E_1 - E_2) + a\bar{H}_{12}(E_3 - E_2) = 0.\\ &\bar{G}_{3v}(E_v - E_3) + \bar{G}_{3w}(E_w - E_3) + a\bar{H}_{18} (E_1 - E_3) + 2a\bar{H}_{12}(E_2 - E_3) = 0, \end{aligned}\right\} \quad (35)$$

or

$$\left.\begin{aligned} &(\bar{G}_{2v} + \bar{G}_{2w} + 2a\bar{H}_{12})(E_2 - E_1) - a\bar{H}_{12}(E_3 - E_1) = \bar{G}_{2v}(E_v - E_1) + \bar{G}_{2w}(E_w - E_1),\\ &-2a\bar{H}_{12}(E_2 - E_1) + (\bar{G}_{3v} + \bar{G}_{3w} + a\bar{H}_{13} + 2a\bar{H}_{12})(E_3 - E_1) = \bar{G}_{3v} (E_v - E_1) + \bar{G}_{3w}(E_w - E_1) \end{aligned}\right\} \quad (36)$$

Calculation of radiant heat transfer is carried out for 1 m length of the duct and the results are given in Table 3. Numerical calculation

Table 2. Initial data for numerical example

	The bottom of the duct	Adiabatic duct walls	Zone of flame	The zone of non-burning gases
No. of zone on Fig. 3	1	2, 3, 4	w	v
Temperature t°C	1000	—	1200	1050
Absorptivity of wall, a	0·8	0·8	—	—
Black-body radiation density, E kW/m²	153·6	—	271·7	176·8

of the values given in Table 3 is illustrated for the following cases:

(1) For calculation of H_{12} we take from Table 1:

$$N_2(0) = 0{\cdot}4244,\ N_2[0{\cdot}364\sqrt{(1+0{\cdot}16)}] = 0{\cdot}2020$$

$$N_2(0{\cdot}364 \times 0{\cdot}4) = 0{\cdot}3146;\ N_2(0{\cdot}364) = 0{\cdot}2121$$

and using formula (24) we have

$$H_{12} = 2 \times 0{\cdot}364\,(0{\cdot}4244 + 0{\cdot}2020 - 0{\cdot}3146 - 0{\cdot}2121) = 0{\cdot}1369 \text{ m}^2/\text{m}.$$

(2) For lower location of the flame we obtain:

$$H_{26} = 0{\cdot}2;\quad H_{28} = 0{\cdot}0909;\quad H_{2\mathrm{III}7} = 0{\cdot}1316;\quad H_{2\mathrm{I}5} = 0;\quad H_{(2\mathrm{I}+2\mathrm{II})7} = 0{\cdot}0694;\quad H_{25} = H_{(2\mathrm{II}+2\mathrm{III})5} = 0{\cdot}1369$$

and from equations (27) and (28)

$$G_{2w} = 0{\cdot}2000 - 0{\cdot}0909 + 0{\cdot}1316 + 0 - 0{\cdot}0694 - 0{\cdot}1369 = 0{\cdot}0344 \text{ m}^2/\text{m}.$$

(3) The wall–wall overall interchange areas are determined from equation system (31):

$$\left.\begin{aligned}
&\bar{H}_{11} - 2\frac{0{\cdot}2}{1}0{\cdot}1369\,\bar{H}_{12} - \frac{0{\cdot}2}{0{\cdot}4}0{\cdot}0487\,\bar{H}_{13} = 0\\
&-\frac{0{\cdot}2}{0{\cdot}4}0{\cdot}1369\,\bar{H}_{11} + \left(1 - \frac{0{\cdot}2}{1}0{\cdot}5378\right)\bar{H}_{12} - \frac{0{\cdot}2}{0{\cdot}4}0{\cdot}1369\,\bar{H}_{13} = 0{\cdot}1369\\
&-\frac{0{\cdot}2}{0{\cdot}4}0{\cdot}0487\,\bar{H}_{11} - 2\frac{0{\cdot}2}{1}0{\cdot}1369\,\bar{H}_{12} + \bar{H}_{13} = 0{\cdot}0487
\end{aligned}\right\}$$

and

$$\left.\begin{aligned}
&\left(1 - \frac{0{\cdot}2}{0{\cdot}4}0{\cdot}0487\right)\bar{H}_{21} - \frac{0{\cdot}2}{1}0{\cdot}1369\,\bar{H}_{22} - \frac{0{\cdot}2}{1}0{\cdot}1369\,\bar{H}_{24} = 0{\cdot}1369\\
&-2\frac{0{\cdot}2}{0{\cdot}4}0{\cdot}1369\,\bar{H}_{21} + \bar{H}_{22} - \frac{0{\cdot}2}{1}0{\cdot}5378\,\bar{H}_{24} = 0.
\end{aligned}\right\}$$

As an example we have:

$$\bar{H}_{11} = \frac{D_{11}}{D} = \frac{0{\cdot}00892}{0{\cdot}8843} = 0{\cdot}0101 \text{ m}^2/\text{m},$$

where

$$D = \begin{vmatrix} +1 & -0{\cdot}05476 & -0{\cdot}02435 \\ -0{\cdot}06845 & +0{\cdot}89244 & -0{\cdot}06845 \\ -0{\cdot}02435 & -0{\cdot}05476 & +1 \end{vmatrix} = 0{\cdot}8843$$

and

$$D_{11} = \begin{vmatrix} +0 & -0{\cdot}05476 & -0{\cdot}02435 \\ +0{\cdot}1369 & +0{\cdot}89244 & -0{\cdot}06845 \\ +0{\cdot}0487 & -0{\cdot}05476 & +1 \end{vmatrix} = 0{\cdot}00892$$

(4). For lower location of the flame, the value of $\bar{G}_{1w}$ is calculated by (33) as follows:

$$G_{1w} = 0{\cdot}0408 + \frac{0{\cdot}8}{0{\cdot}4}0{\cdot}0101 \times 0{\cdot}408 + 2\frac{0{\cdot}8}{1}0{\cdot}1586 \times 0{\cdot}0344 + \frac{0{\cdot}8}{0{\cdot}4}0{\cdot}0576 \times 0{\cdot}0057 = 0{\cdot}0434 \text{ m}^2/\text{m}.$$

Table 3. The calculation of the radiant heat exchange in the furnace chamber

Title	Symbol	Dimension	No. of the formula	Value by the flame location	
				lower	upper
1. The values of wall–wall direct interchange areas	H_{13}	m²/m	(23)	0·0487	0·0487
	H_{15}			0·4000	0·0762
	H_{17}			0·2204	0·0560
	H_{24}			0·5378	0·5378
	H_{26}, H_{48}			0·2000	0·2000
	H_{28}, H_{46}			0·0909	0·1059
	H_{35}			0·0487	0·1702
	H_{37}			0·0652	0·2936
	H_{12}, H_{14}, H_{23}, H_{34}	m²/m	(24)	0·1369	0·1369
	H_{16}, H_{18}			0·0694	0·0076
	H_{2I5}, H_{4I5}			0	0·1272
	$H_{(2I+I2I)7}$, $H_{(4I+4II)7}$			0·0694	0·1348
	$H_{(2II+2III)5}$, $H_{(4II+4III)5}$			0·1369	0·0903
	H_{2III7}, H_{4III7}			0·1316	0·0396
	H_{36}, H_{38}			0·0054	0·0470
2. The values of volume–wall direct interchange areas	G_{1w}	m²/m	(28)	0·0408	0·0050
	G_{2w}, G_{4w}			0·0344	0·0358
	G_{3w}			0·0057	0·0294
	G_{1v}	m²/m	(29)	0·0366	0·0725
	G_{2v}, G_{4v}			0·1540	0·1526
	G_{3v}			0·0718	0·0481
3. The values of wall–wall overall interchange areas	$\bar{H}_{11}$, $\bar{H}_{33}$	m²/m	(31)	0·0101	0·0101
	$\bar{H}_{12}$, $\bar{H}_{14}$, $\bar{H}_{23}$, $\bar{H}_{34}$			0·1586	0·1586
	$\bar{H}_{13}$			0·0576	0·0576
	$\bar{H}_{22}$, $\bar{H}_{44}$			0·0827	0·0827
	$\bar{H}_{24}$			0·5673	0·5673
4. The values of volume–wall overall interchange areas	$\bar{G}_{1w}$	m²/m	(33)	0·0434	0·0082
	$\bar{G}_{2w}$, $\bar{G}_{4w}$			0·0426	0·0432
	$\bar{G}_{3w}$			0·0091	0·0320
	$\bar{G}_{1v}$			0·0486	0·0840
	$\bar{G}_{2v}$, $\bar{G}_{4v}$			0·1826	0·1820
	$\bar{G}_{3v}$			0·0830	0·0601
5. The differences of black-body energy emissions	$E_2 - E_1$	kW/m²	(36)	25·8	27·7
	$E_3 - E_1$			24·4	31·2
6. The resulting heat flux	$Q_{res,1}$	kW/m	(34)	11·1	9·1

(5). In case of lower location of the flame equation system (36) is as:

$$+0{\cdot}47900(E_2 - E_1) - 0{\cdot}1269(E_3 - E_1) = \\ = 9{\cdot}267$$

$$-0{\cdot}2538(E_2 - E_1) + 0{\cdot}3919(E_3 - E_1) = \\ = 3{\cdot}000$$

and it is found that $E_2 - E_1 = 25{\cdot}8$ kW/m² and $E_3 - E_1 = 24{\cdot}4$ kW/m². Using these values, the resulting heat flux is calculated from formula

Table 4. A comparison between the numerical example and the experimental data

No.	Character	Dimension	Numerical example	Experimental data
1	Length of the furnace chamber	m	∞	0·7
2	Absorbitivity of the walls and bottom	—	0·8	unknown
3	Temperature of the bottom	°C	1000	unknown, can be estimated to 950–1050
4	Heat flux at the walls	—	walls are considered to be as adiabatic surfaces	unknown
5	Temperature field in the chamber	—	two discrete zones	continuous, having a peak as a flame zone
6	Temperature of the flame zone	°C	1200	1100–1250
7	Temperature of non-burning gaseous zone	°C	1050	1010–1100
8	Resulting heat flux at the bottom reduced to the 1 m length of the chamber:			
	at lower location of the flame	kW/m	11·1	10·5
	at upper location of the flame	kW/m	9·1	8·8
9	Difference of heat transfer at various locations of the flame	per cent	22	19

(34), as

$$Q_{res,1} = 0{\cdot}8\,(23{\cdot}2 \times 0{\cdot}0486 + 118{\cdot}1 \times 0{\cdot}0434 + 2 \times 25{\cdot}8 \times 0{\cdot}1586 \times 0{\cdot}8 + 0{\cdot}8 \times 24{\cdot}4 \times 0{\cdot}0576) = 11{\cdot}1 \text{ kW/m.}$$

DISCUSSION

It is of interest to compare the results of the calculation method with the real conditions. With this in mind, the initial data of the numerical example were chosen to fit the experimental data of Zahkarikov [12, 13]. However, it was necessary to choose some approximate data in the numerical example, since not all factors were determined in experiments. Moreover the calculation would become rather complicated if we accounted for the less important factors.

In Table 4 results and some initial data of the numerical example are compared with those of the experiment. The numerical calculation can be seen to yield satisfactory results. It should be noted that absolute values of resulting radiant flux decrease rapidly if some lower value for the temperature of the volume zones is chosen. The numerical calculation shows that when the flame temperature is 1180°C, the resulting flux at the lower location of the flame is 10·2 kW/m and at the higher location of the flame it is only 8·5 kW/m. Decreases in relative heat transfer at various locations of the flame vary between 20 and 22 per cent. This result can be used to explain the experimental fact, that at higher loads of the furnace chamber, the mentioned difference increases [12, 13].

The strong dependence of heat transfer on the temperature conditions mitigates against the attainment of high accuracy in the resulting heat flux. This method can be recommended only for approximate calculation. In this connexion increasing the number of zones is not effective because current methods do not allow of the use of the temperature distribution in the medium as initial data.

However, being somewhat approximate, the method is effective in solving problems where relative heat-transfer conditions are studied, for example in problems of heat transfer at various locations of the flame in the furnace chamber, and in problems where resulting heat flux distribution on the walls of the furnace is to be determined. It follows from the above example, that the method can give approximate results even in case of very short ducts.

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Abstract—An approximate radiant heat-transfer calculation method of dealing with cylindrical systems of rectangular cross section is presented in the paper. Under given conditions, the two-dimensional theory may be applied. A system of gray walls is considered, filled with an absorbing medium having no reflectivity and no refractivity. The method of solution of the problem is based on integral equations. Calculation can be carried out in the following way:

1. The system is divided into zones.
2. For a duct of rectangular cross section, having four surface zones and two volume zones, direct interchange factors and areas are determined by (21) to (29).
3. Determination of overall interchange areas and factors is carried out by using (30) to (33).
4. Calculation of resulting heat fluxes or zone temperatures is carried out using (17) and (18).

The method is illustrated by an example. The numerical part of the example is given in Table 3, whilst in Table 4 some experimental data are compared with initial conditions and results of the numerical example. The method can be used for solving problems on radiant heat transfer if various locations of the flame in the furnace chamber are to be taken into consideration. The method may also be useful in determining resulting heat flux distribution in a cross section of the furnace.

Résumé—Une méthode de calcul approximatif du rayonnement de la chaleur dans un systéme cylindrique ayant un profil rectangulaire est présentée.

Dans des conditions données on peut employer une théorie bidimensionnelle. Un systéme à surfaces grises, rempli d'un milieu qui absorbe, mais ne refléte et refracte le rayonnement, est étudié. La méthode de la solution du probléme est fondée sur les équations integrales. Le calcul séffectue dans l'ordre suivant:

(1) On partage en zones.

(2) Dans le cas d'un canal rectangulaire, dint chaque paroi est prise pour une zone superficielle et dont le volume est devisé en deux zones, les facteurs et les surfaces d'échange de chaleur directes sont calculés à l'aide des formules (21)–(29).

(3) Les facteurs et les surfaces d'échange résultants sont calculés à l'aide des formules (30)–(33).

(4) On calcule la quantite résultante de la chaleur transmise (ou les témperatures zonales) d'aprés les formules (17) et (18).

La méthode est illustrée d'un exemple. La part numérique de l'exemple est donnée dans la table 3; dans la table 4 sont quelques données expérimentales comparées aux données initiales et aux résultats obtenus de l'exemple. La méthode peut étre employée pour résoudre les problémes de foyer compte tenu de la position de la flamme. La méthode peut étre employée également pour déterminer la courbe de distribution d'échange de chaleur dans les coupes transversales du foyer.

Zusammenfassung—Eine Methode zur annähernden Berechnung der Wärmestrahlung in einem zylindrischen System mit rechtwinkligem Querschnitt ist präsentiert. In gegebenen Bedingungen kann eine zweidimensionale Theorie angewendet werden. Ein System mit grauen Wandungen und einem absorbierenden, doch nicht reflektierenden und refraktierenden Medium ist in Betracht gezogen. Die Lösung des Problems wird durch Integralgleichungen erhalten. Die Berechnung kann in folgender Ordnung durchgeführt werden:

(1) Das System ist in Zonen zu verteilen.

(2) Für den Kanal mit rechtwinkligen Querschnitt, mit 4 Flächen- und 2 Raumzonen sind die Faktoren und Flächen des direkten Wärmeübergangs gemäss den Formeln (21)–(29) zu berechenen.

(3) Die resultierenden Übergangsfaktoren und -Flächen werden nach den Formeln (30)–(33) berechnet.

(4) Man berechnet die resultierenden übertragenen Wärmemengen (oder dir Zonentemperaturen) nach den Formeln (17) und (18).

Die Methode ist mit einem Beispiel illustriert. Der numerische Teil des Beispiels ist in Zahlentafel 3 angegeben, während in Zahlentafel 4 einige Experimentaldaten mit den Ausgangsdaten und Ergebnissen des Beispiels vergleichen werden. Die Methode ist bei Feuerraumproblemen anwendbar, wenn die Lage der Flamme berücksichtingt wird. Auch zur Bestimmun der Verteilungskurve des Wärmeübergangs in Feuerraumquerschnitten lässt sich die Methode vorteilhaft anwenden.